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Solution – Joe Blitzstein (One of my favourite stats professor) has said that ‘thinking conditionally is a condition for thinking correctly’. The concept of conditional probability is all about applying condition to a known case to find something that is not known obviously.

The first part of the question asks to calculate conditional probabilities. a1and a2 are categorical variables so all we need to do is count. a3 is numerical and the question asks us to use normal probability distribution function. So, we will calculate mean and variance that are required to calculate these probabilities so we can use them when needed.

The first of which is, P(a1|Y)

It means the probability of a1 to be equal to something given we know it is from the class ‘Y’.

Now, a1 can either by T and F.

So, P(a1 = T|Y) = just count for all examples with class = Y, how many has a1 = T. There are five examples with class Y. Out of them, 4 has a1 = T

So, P(a1 = T|Y) = 4/5 = 0.8

Similarly, P(a1 = F|Y) = 0.2

P(a1 = F|N) = 5/4 = 0.8

P(a1 = T|N) = 1/4 = 0.2

Now, Similarly for P(a2|Y) and P(a2|N)

P(a2 = T | Y) = 2/5 = 0.4

P(a2 = F | Y) = 3/5 = 0.6

P(a2 = T | N) = 3/5 = 0.6

P(a2 = F | N) = 2/5 = 0.4

Now, Similarly for P(a3|Y)

There are 5 points with Y = 5. For them, a3 = (1, 4, 5, 7, 8)

Mean = 5

s.d. = 2.45

s.d^2 = 6

For Class N, there are 5 points with a3 = (3, 4, 5, 6, 7)

Mean = 5

s.d. = 1.414

s.d^2 = 2

Now moving to next question.

We need to calculate class probabilities. Class probabilities are 0.5 each as both classes are equally representative.

No the point for which prediction has to be done is (T, F, 4).

So, we have to find, “what is the probability that the class of a point is Y, given that its attributes are, a1 = T, a2 = F, and a3 = 4?

Simply, multiply probabilities of a1 = T for Y (0.8), a2 = F for Y (0.6). and a3 = 3 for Y (yet to be calculated) with the class probability of Y (0.5)

P(a3 = 3| Class = Y) = use that big formula. And put mean = 5, s.d. = 2.45 and x = 3

= 0.116 (<https://keisan.casio.com/exec/system/1180573188> use this. Just enter values)

So, final P(Y | x = (T, F, 3)) = 0.5 x 0.8 x 0.6 x 0.116 = 0.02784

Similarly, repeat calculations of class = N. we already have all the data.

Simply, multiply probabilities of a1 = T for N (0.2), a2 = F for N (0.4). and a3 = 3 for Y (yet to be calculated) with the class probability of Y (0.5)

First find out P(a3 = 3 | class = N) using that online calculator. This time, put mean = 5 and s.d. = 1.414

The answer is 0.103

Now multiply it all. 0.5 x 0.2 x 0.4 x 0.103 = 0.004

So, the final verdict is that the point is more likely to belong to ‘Y’ class